Pion Form Factor and Ambiguities in a Renormalizable Version of the Nambu-Jona-Lasinio Model[*]

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We analyze the presence of an ambiguity in the pion electromagnetic form factor within a renormalizable version of the Nambu-Jona-Lasinio model. We found out that the ambiguity present on the evaluation of the form factor decouples from its transversal part, confirming previous results obtained ignoring the ambiguity. This result helps us to understand the role played by finite but undetermined quantities in Quantum Field Theories.

I. INTRODUCTION

Recently ([1], [2], [3]) it was brought to attention the existence of finite but undetermined radiative corrections in Quantum Field Theory. These finite amplitudes are related to differences between divergent quantities of the same degree of divergence, and the result obtained by employing a particular regularization scheme can be different of the result obtained by another one (in particular, this difference appears between gauge invariant and non invariant schemes) [4]. This situation becomes more complex in models with parity violating quantities, where dimensional regularization is inappropriate. A classical example is the ABJ anomaly [5], where one must choose between the transversality of the vector or axial-vector currents. At this point, the ambiguity plays a crucial role in determining which one of the symmetry relations will be violated.

Scarpelli et al. [6] studied this situation employing a regularization independent procedure that have been called Implicit Regularization (IR) ([7], [8]). By using IR, they had shown that the same ambiguities can occur in several situations, as in QED vacuum polarization tensor. In this case, gauge invariance has to be used to fix the ambiguity value. As proposed in [3], the ambiguity was fixed in order to preserve some physical relationship, e.g., symmetry. One of the advantages of the IR procedure is that the amplitudes can be evaluated almost all the time without specifying some particular regularization scheme. Of course, particular attention must be paid to regularization dependent quantities. Another procedure that have been employed based on the same principles is the differential regularization [9], where the amplitudes are evaluated in configuration space. For our purposes here, i.e., evaluate the pion form factor in a regularization independent way in energy-momentum space, IR is more appropriate.

Non-renormalizable models do not show the presence of ambiguities, in the sense that the regularization scheme employed is part of the model. This is the case of the Nambu-Jona-Lasinio (NJL) model ([10], [11]), and one can say that Pauli-Villars regularized NJL model is different of the sharp covariant cut-off regularized NJL model. Once defined which regularization scheme will be used on the amplitudes evaluation of the NJL model, also the result of the differences between divergent quantities will be defined. Thus, there are no reasons to leave the ambiguity undetermined in such models. In general, gauge invariant regularization schemes are used to treat the regularized NJL model, but regularization schemes that destroy the quadratic divergence, necessary to do the correct adjustment of the model to the phenomenology, must be avoided. This is the case of the usual gauge invariant Pauli-Villars regularization, and some modifications of the scheme must be done in order to preserve the quadratic divergence and symmetries of the model [12].

But in renormalizable extensions of the NJL model ([13], [14]) the regularization scheme is not part of the model, and thus these models can present ambiguities. Due its relative simplicity, the renormalizable extension of the scalar/pseudo-scalar section of the NJL SU(2) model provides a good scenario where one can study the role played by ambiguities in a QFT: (i) it is renormalizable; (ii) it is a fermionic model; (iii) it presents chiral symmetry, in the vanishing fermion mass limit; (iv) it presents parity violating couplings; (v) it presents ambiguities; (vi) there are experimental data available to check its results. Of course there are another models whose fulfill some of the characteristics listed above, such as the chiral Schwinger model, the Gross-Neveu model, and so on.

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We will adopt here the renormalizable extension of the NJL model presented in [14]. This renormalizable non-trivial extension is constructed by using a mean field expansion [15] that results in a renormalizable but trivial effective Lagrangean [16]. The first-order effective Lagrangean presents kinetic and mesonic interaction terms radiatively generated, and by augmenting the original effective lagrangean with similar terms, one can avoid the non-renormalizability and triviality of the model [14]. This procedure, of course, leads to a model that is different from the original regularized NJL model, but that is related to the latter. In this condition, results are formally identical to the results presented by the NJL model, but without any explicit cut-off dependence (the connection limit can be taken), leaving the model regularization scheme's independent.

Both regularized and renormalized versions of the NJL model are appropriated to the study of mesonic properties, and reproduces observables with a reasonable agreement with experimental data. As discussed before, there is not ambiguities in regularized NJL model, since they are fixed by the choice of an specific regularization scheme. But in the renormalizable extension of the NJL model, these ambiguities are present on the calculation of several processes. The results presented in [14] do not took in account the influence of these ambiguities, they were fixed by implicitly employing a gauge invariant regularization scheme that fixes them to zero. In this letter, in particular, we will evaluate the pion electromagnetic form factor within the renormalizable extension of the NJL model employing IR and, following the prescription suggested in [3], leaving the ambiguity undetermined. We will show that the ambiguity present on the pion form factor cannot be fixed by symmetry relationships (Ward Identity) and does not affect the previsibility of the model.

II. THE PION CHARGE FORM FACTOR

In the scalar/pseudo-scalar sector of the NJL model, the pion form factor is obtained by evaluating the diagram depicted in figure 1. There is a whole contribution of the vector/axial-vector sector that reproduces the influence of the vector mesons (Vector Dominance Model - VDM) that is missing in the present analysis. In the time-like region the contribution coming from these vector mesons is very important, so we will restrict our analysis to the space-like region. We must also remark that our interest is to study the presence of ambiguities in this process, and thus our results will not be jeopardized by not employing the VDM.

FIG. 1. Feynman diagram for the pion charge form factor in the NJL model.

The amplitude correspondent to the diagram showed in figure 1 is given by

$$-i\Gamma^\mu(p,p') = N_c N_f\, g_{\pi qq}^2 \int \frac{d^4k}{(2\pi)^4} Tr\{\gamma^5 \frac{i}{(\not k - \not p) - m}\gamma^5 \frac{i}{\not k - m}\gamma^\mu \frac{i}{(\not k - \not q) - m}\}. \tag{1}$$

with $q = p + p'$.

Evaluating (1) by using IR corresponds to isolate, without the use of any explicit regularization, the divergences of the amplitude in terms that are independent of the external momentum. As remarked in [6], we must not make use of symmetrical integration. By proceeding in this way, we obtain (see notes on Appendix)

$$\Gamma^\mu = 2N_C g_{\pi qq}^2(p^\mu - p'^\mu)\frac{1}{4N_C g_\pi^2} + 2N_C g_{\pi qq}^2 m_\pi^2 (p^\mu - p'^\mu)\left(Z_0'(m_\pi^2) - I_3(q^2, m_\pi^2, m_\pi^2)\right) \tag{2}$$
$$- 2N_C g_{\pi qq}^2 q_\nu Y^{\mu\nu},$$

where

$$Z_0(p^2) = \frac{1}{16\pi^2}\int_0^1 dz \ell n\left[1 - \frac{p^2}{m^2}z(1-z)\right] \tag{3}$$
$$= -\frac{2i}{(4\pi)^2}\{-1 + \frac{\sqrt{p^4 - 4p^2 m^2}}{p^2}\arctan h(\frac{p^2}{\sqrt{p^4 - 4p^2 m^2}})\}$$

$$Z_0'(p^2) = \frac{dZ_0(p^2)}{dp^2}, \tag{4}$$

and

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$$I_3(p,q) = i \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m^2)[(k-p)^2 - m^2][(k-q)^2 - m^2]}. \tag{5}$$

At this point, we identify the ambiguity in (2) by the term

$$Y^{\mu\nu} = 2 \int \frac{d^4k}{(2\pi)^4} \frac{k^\mu k^\nu}{(k^2 - m^2)^3} - g^{\mu\nu} \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m^2)}. \tag{6}$$

It is a simple matter to evaluate the quantity in (6) in different regularization schemes. So, in dimensional regularization or gauge invariant Pauli-Villars one can obtain $Y^{\mu v} = 0$, but in covariant sharp cut-off one can obtain $Y^{\mu\nu} = \frac{ig^{\mu\nu}}{2(4\pi)^2}$. Since we are not specifying here any explicit regularization scheme, we will adopt the procedure suggested in [3] and leave this ambiguity undetermined up to the end of the evaluation. Setting, by using Lorentz invariance,

$$Y^{\mu\nu} = g^{\mu\nu}\alpha \tag{7}$$

we can write (2) as

$$\begin{aligned} \Gamma^\mu &= 2N_C g_{\pi qq}^2 (p^\mu - p'^\mu) \frac{1}{4N_C g_\pi^2} \\ &+ 2N_C g_{\pi qq}^2 m_\pi^2 (p^\mu - p'^\mu) \left(Z_0'(m_\pi^2) - I_3(p^2, m_\pi^2, m_\pi^2) \right) \\ &- 2N_C g_{\pi qq}^2 q^\mu \alpha. \end{aligned} \tag{8}$$

The divergent terms present in (8) were absorbed in the contra-terms of the model, since it is renormalizable, in the same way presented in [14], resulting in renormalized parameters g_π and $g_{\pi qq}$.

III. THE AMBIGUITY

Equation (8) shows us that the ambiguity belongs to the longitudinal term of the amplitude Γ^μ, the transverse part being free of ambiguities. In order to verify if symmetries (such as gauge invariance) fix the ambiguity, we must verify the Ward identity related to the pion-photon vertex function, that in the NJL model reads

$$q_\mu \Gamma^\mu = g_{\pi qq}^2 (T^{\pi\pi}(p,q) - T^{\pi\pi}(p)) \tag{9}$$

where

$$T^{\pi\pi}(p,q) = \int \frac{d^4k}{(2\pi)^4} Tr\{\gamma_5 \frac{1}{\not{k} - \not{p} - m} \gamma_5 \frac{1}{\not{k} - \not{q} - m}\}$$

and

$$T^{\pi\pi}(p) = \int \frac{d^4k}{(2\pi)^4} Tr\{\gamma_5 \frac{1}{\not{k} - m} \gamma_5 \frac{1}{\not{k} - \not{p} - m}\}.$$

In the left hand side of eq. (9), the product of the photon momentum q_μ by the transverse part of Γ^μ vanishes. Thus, only the ambiguous part of Γ^μ survives, leading to

$$2N_C g_{\pi qq}^2 q^2 \alpha = g_{\pi qq}^2 (T^{\pi\pi}(p,q) - T^{\pi\pi}(p)) \tag{10}$$

The right hand side of eq. (10) can be explicitly evaluated, and, also applying IR, the Ward identity (9) can be exactly verified. Thus, the Ward identity (9) is satisfied even in the presence of the ambiguity (6), and does not fix its value. This ensures charge conservation as a feature independent of the presence of the ambiguity.

The next question to be answered is how the ambiguity present in the pion electromagnetic vertex function Γ^μ affects the pion form factor. Let us recall that the amplitude corresponding to figure 1 can be obtained by

$$M = \lim_{p^2, p'^2 = m_\pi^2} \varepsilon_{1\mu} \Gamma^\mu(p, p') \tag{11}$$

where $\varepsilon_{1\mu}$ is the polarization vector of the off-shell photon with momentum $q = p + p'$. Since $\varepsilon_{1\mu}$ is transverse to the photon momentum, the longitudinal part of Γ^μ will not be present in the amplitude. Thus, this amplitude and, in consequence, the pion charge form factor, will be free from the ambiguity present in $\Gamma^\mu(p, p')$.

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IV. NUMERICAL RESULTS

Although the ambiguity does not affect the pion form factor, we present here the numerical results obtained for the pion form factor within the renormalizable extension of the NJL model. The results and the parameters set presented here are the same as in [14], we reproduce these results for reasons of completeness. We apply implicit regularization, and fit the following sets of parameters: ($m = 350 MeV$, $g_\pi = 3.752$ and $\mu_\pi = 141 MeV$) and ($m = 210 MeV$, $g_\pi = 2.25$ and $\mu_\pi = 141 MeV$). The parameters are adjusted to reproduce $f_\pi = 93.3 MeV$ and $m_\pi = 139 MeV$. The comparison between the results of the renormalizable extension of the NJL model and experimental data are shown in figure 2. The electromagnetic radius of the pion can be obtained by

$$< r_\pi^2 >= -6 \frac{dF_\pi(q^2)}{dq^2}|_{q^2=0} \tag{12}$$

and, for the sets of parameters above we obtain, respectively

$$< r_\pi^2 >= 0.58 fm; \quad < r_\pi^2 >= 0.6 fm \tag{13}$$

that are to be compared with the experimental result [17]

$$< r_\pi^2 >_{\text{exp}} = 0.678 \pm 0.012 fm \tag{14}$$

We observe that a better fit is obtained with the lower constituent quark mass, $m = 210 MeV$. In fact, if the constituent quark mass is lowered, the fit with the experimental data is improved. But, as stated before, the model studied here does not include the vector/axial-vector meson sector, and a complete agreement with experiment is not to be expected.

FIG. 2. Pion charge form factor in the space–like region. The results obtained from the renormalizable extension of the NJL model are presented for $m = 210 MeV$ (dotted line) and $m = 350 MeV$ (solid line) and are compared to experimental data [17].

V. CONCLUSION

In summary, we obtained the pion charge form factor within a renormalizable extension of the SU(2) NJL model with scalar/pseudo-scalar couplings. We had shown that, in this model, the pion-photon vertex function contains an ambiguous term, i.e., a finite but regularization dependent term. We verify that, even in the presence of the ambiguity, the Ward identity related to this vertex function is satisfied, so it does not fix the ambiguity value. Also, in the amplitude related to this process, and consequently in the pion charge form factor, the ambiguity is not present, since it is transverse to the photon polarization vector, and thus decouples from the physical content of the model.

This provides an example of one of the roles played by ambiguities in Quantum Field Theory: in previous works, it was shown that ambiguities can be fixed either by symmetry relationships (as in QED [6]) or by phenomenology (as in the neutral pion electromagnetic decay or in proton decay [3]). Here, we found out that there is a third situation, where the ambiguity cannot be fixed by symmetry, but completely decouples from the physical content of the model, remaining independent from the regularization scheme employed.

VI. ACKNOWLEDGMENTS

VII. APPENDIX

In order to show explicitly how we isolated the ambiguity in the pion charge form factor, we will proceed, in this appendix, the computation of the ambiguous part of (1). After taking the traces on color, flavor and Dirac spaces, we obtain the following two ambiguous integrals

$$\xi^{2\mu} = \int \frac{d^4k}{(2\pi)^4} \frac{k^2 k^\mu}{(k^2 - m^2)[(k-p)^2 - m^2][(k-q)^2 - m^2]}, \tag{A1}$$

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and

$$p_\mu \xi^{\mu\nu} = p_\mu \int \frac{d^4k}{(2\pi)^4} \frac{k^\mu k^\nu}{(k^2 - m^2)[(k-p)^2 - m^2][(k-q)^2 - m^2]}. \tag{A2}$$

By adding and subtracting a $m^2 k^\mu$ term on the numerator of (A1), we obtain

$$\begin{aligned} \xi^{2\mu} &= \int \frac{d^4k}{(2\pi)^4} \frac{k^\mu}{[(k-p)^2 - m^2][(k-q)^2 - m^2]} \\ &+ m^2 \int \frac{d^4k}{(2\pi)^4} \frac{k^\mu}{(k^2 - m^2)[(k-p)^2 - m^2][(k-q)^2 - m^2]} \\ &= I_t^\mu + m^2 \xi^\mu \end{aligned} \tag{A3}$$

The last integral in (A3) is finite, and can be evaluated directly, yielding

$$\xi^\mu = (p^\mu - p'^\mu)(Z_0'(m_\pi^2) - I_3(q^2, m_\pi^2, m_\pi^2)) \tag{A4}$$

Introducing one Feynman parameter on I_t^μ, we obtain

$$I_t^\mu = \int_0^1 dx \int \frac{d^4k}{(2\pi)^4} \frac{k^\mu}{[(k-t)^2 - M^2]^2} \tag{A5}$$

where

$$M^2 = m^2 - (p-q)^2 x(1-x), \tag{A6}$$

and

$$t^\mu = p^\mu x + q^\mu (1-x). \tag{A7}$$

By adding and subtracting a t^μ term in the numerator of (A5), we obtain

$$I_t^\mu = \int_0^1 dx \left\{ \int \frac{d^4k}{(2\pi)^4} \frac{k^\mu - t^\mu}{[(k-t)^2 - M^2]^2} + t^\mu \int \frac{d^4k}{(2\pi)^4} \frac{1}{[(k-t)^2 - M^2]^2} \right\} \tag{A8}$$

The last integral in (A8) is logarithmically divergent, an we can safely proceed a shift in the integration momentum $k - t \to k'$. More careful must be taken in the evaluation of the first integral in (A8), since it is linearly divergent. To properly isolate the ambiguity term present in this integral, we introduce the translation operator $e^{-t^\mu \frac{\partial}{\partial k^\mu}}$ and rewrite (A8) as

$$I_t^\mu = \int_0^1 dx \left\{ \int \frac{d^4k}{(2\pi)^4} e^{-t^\mu \frac{\partial}{\partial k^\mu}} \left\{ \frac{k^\mu}{(k^2 - M^2)^2} \right\} + t^\mu \int \frac{d^4k}{(2\pi)^4} \frac{1}{[k^2 - M^2]^2} \right\}. \tag{A9}$$

Expanding the translation operator, we find

$$\begin{aligned} \int \frac{d^4k}{(2\pi)^4} e^{-t^\mu \frac{\partial}{\partial k^\mu}} \left\{ \frac{k^\mu}{(k^2 - M^2)^2} \right\} &= \int \frac{d^4k}{(2\pi)^4} \frac{k^\mu}{(k^2 - M^2)^2} - t^\nu \int \frac{d^4k}{(2\pi)^4} \frac{\partial}{\partial k^\nu} \left\{ \frac{k^\mu}{(k^2 - M^2)^2} \right\} \\ &+ \frac{t^\nu t^\rho}{2} \int \frac{d^4k}{(2\pi)^4} \frac{\partial^2}{\partial k^\nu \partial k^\rho} \left\{ \frac{k^\mu}{(k^2 - M^2)^2} \right\} + \dots \end{aligned} \tag{A10}$$

The integrals with odd terms in its integrands in (A10) vanish, and all the terms with derivatives of order greater than 2 will also vanish. We obtain

$$I_t^\mu = \int_0^1 dx \left\{ \int \frac{d^4k}{(2\pi)^4} \frac{-2g^{\mu\nu}}{(k^2 - M^2)^2} + \int \frac{d^4k}{(2\pi)^4} \frac{8k^\mu k^\nu}{(k^2 - M^2)^3} \right\} + t^\mu \int \frac{d^4k}{(2\pi)^4} \frac{1}{[k^2 - M^2]^2}. \tag{A11}$$

We identify the terms in brackets in (A11) as the ambiguous part of $\xi^{2\mu}$. Nevertheless, there is still a dependence on the integrand of (A11) on p, q and x, and one can argue if there is some finite contribution coming from (A11).

We remark that this term is ambiguous, and the sum of an ambiguity with any finite non-ambiguous term will remain ambiguous. Although this fact, one can find, after successively applying the following relationship

$$\frac{1}{k^2 - M^2} = \frac{1}{k^2 - m^2} + \frac{M^2 - m^2}{(k^2 - m^2)(k^2 - M^2)} \tag{A12}$$

and some straightforward calculations, that

$$\int_0^1 dx \left\{ \int \frac{d^4k}{(2\pi)^4} \frac{-2g^{\mu\nu}}{(k^2 - M^2)^2} + \int \frac{d^4k}{(2\pi)^4} \frac{8k^\mu k^\nu}{(k^2 - M^2)^3} \right\}$$
$$= \int \frac{d^4k}{(2\pi)^4} \frac{-2g^{\mu\nu}}{(k^2 - m^2)^2} + \int \frac{d^4k}{(2\pi)^4} \frac{8k^\mu k^\nu}{(k^2 - m^2)^3} = 4Y^{\mu\nu}. \tag{A13}$$

The evaluation of $\xi^{\mu\nu}$ follows almost the same steps we follow in the evaluation of $\xi^{2\mu}$. Since it is contracted with the momentum p, we made use of

$$k.p = \frac{1}{2}[(k^2 - m^2) + p^2 + ((k-p)^2 - m^2)] \tag{A10}$$

and obtain

$$p_\mu \xi^{\mu\nu} = \frac{1}{2}\{p^2 \xi^\mu + I_t^\mu - I_q^\mu\}, \tag{A11}$$

where ξ^μ is given by (A4), I_t^μ is given by (A13) and

$$I_q^\mu = \int \frac{d^4k}{(2\pi)^4} \frac{k^\mu}{(k^2 - m^2)[(k-q)^2 - m^2]}, \tag{A12}$$

can be computed in the same way we proceeded on the computation of I_t^μ.

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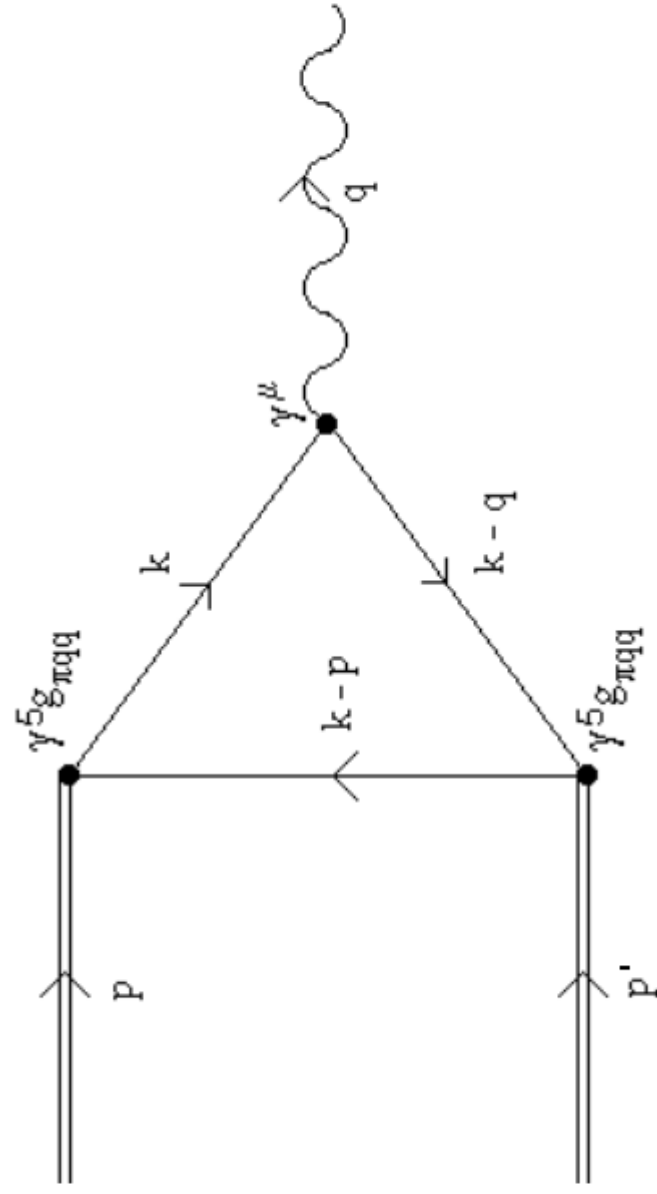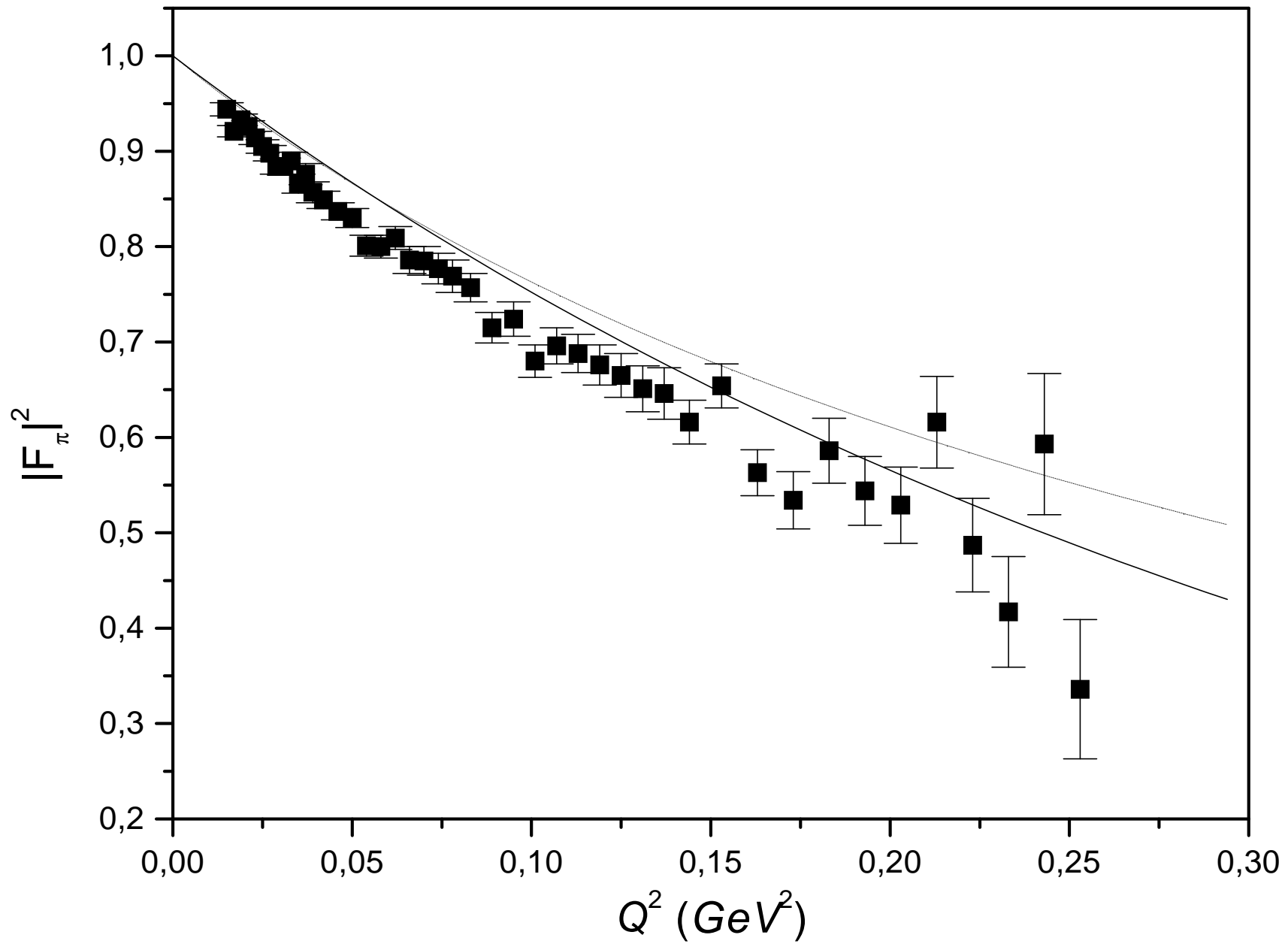[17] S. R. Amendolia et al., Phys. Lett B 178, 435 (1986).

Figure 1

Figure 2